1	b	$\frac{3}{2-\sqrt{y}} \times \frac{2+\sqrt{y}}{2+\sqrt{y}}$ **or** $6+3\sqrt{y}$ **or** $4-y$			M1	for multiplying numerator and denominator by $(2+\sqrt{y})$ **or** a correct expression for the numerator or denominator
			$\frac{6+3\sqrt{y}}{4-y}$	2	A1	

2	(a)	$6\times6+6\times2\sqrt{12}+6\times2\sqrt{12}+\left(2\times\sqrt{12}\right)^2$ or $36+12\sqrt{12}+12\sqrt{12}+4\sqrt{12}\sqrt{12}$ or $36+12\sqrt{12}+12\sqrt{12}+(4\times12)$ or $36+24\sqrt{3}+24\sqrt{3}+48$ or $36+2\times24\sqrt{3}+48$ or $36+6\times2\times2\sqrt{12}+48$		3	M1 for correct expansion of brackets showing **four** terms (need not be simplified) **or** for the use of $(a+b)^2=a^2+2ab+b^2$ **or** for showing or stating $\sqrt{12}=2\sqrt{3}$ oe
		$84+48\sqrt{3}$			M1 (dep on M1)
			Shown		A1 for fully correct working leading to given expression

3	$\frac{18}{\sqrt{7}+1}\times\frac{\sqrt{7}-1}{\sqrt{7}-1}$		3	M1	for $\frac{18}{\sqrt{7}+1}\times\frac{\sqrt{7}-1}{\sqrt{7}-1}$
	eg $\frac{18(\sqrt{7}-1)}{7-1}$			M1	Dep on M1 for a correct numerator **and** multiplying out the denominator to 7 – 1 or 6
	$3\sqrt{7}-3$	$3\sqrt{7}-3$		A1	Dep on M2 Allow $3\left(\sqrt{7}-1\right)$
					Total 3 marks

4	b	E.g. $\frac{2}{6-3\sqrt{2}}\times\frac{6+3\sqrt{2}}{6+3\sqrt{2}}$ or $\frac{2}{6-3\sqrt{2}}\times\frac{-6-3\sqrt{2}}{-6-3\sqrt{2}}$		3	M1	for rationalising the denominator by multiplying numerator and denominator by $6+3\sqrt{2}$ (or $-6-3\sqrt{2}$)
		$\frac{12+6\sqrt{2}}{36-18\sqrt{2}+18\sqrt{2}-18}$ or $\frac{12+6\sqrt{2}}{18}$ or $\frac{12+6\sqrt{2}}{6^2-\left(3\sqrt{2}\right)^2}$ or $\frac{12+6\sqrt{2}}{6^2-9\times2}$			M1	(numerator may be expanded or denominator may be 4 terms which need to be all correct)
			$\frac{2+\sqrt{2}}{3}$		A1	or for stating $a=2$ and $b=3$ for $\frac{2+\sqrt{2}}{3}$ from correct working dep on M2

5	e.g. $40+8\sqrt{x}-5\sqrt{x}-\sqrt{x}\sqrt{x}$ **or** $40+8\sqrt{x}-5\sqrt{x}-\left(\sqrt{x}\right)^2$ **or** $40+8\sqrt{x}-5\sqrt{x}-x$ **or** $40+3\sqrt{x}-x$		3	M1	for a correct expansion with at least 3 out of 4 terms correct oe or all 3 terms correct
		$x=19$		A1	(dep on M1) for $x=19$
		$y=3$		B1	for $y=3$
					Total 3 marks

6	$(\sqrt{2}-1)^2=2-\sqrt{2}-\sqrt{2}+1\left(=3-2\sqrt{2}\right)$	$\frac{(3+\sqrt{8})}{(\sqrt{2}-1)^2}\times\frac{(\sqrt{2}+1)^2}{(\sqrt{2}+1)^2}$		4	M1	expand the denominator (accept $2-2\sqrt{2}+1$ - must see expansion) **OR** method to rationalise using $(\sqrt{2}+1)^2$
	$\frac{\left(3+\sqrt{8}\right)}{"\left(3-2\sqrt{2}\right)"}\times\frac{\left(3+2\sqrt{2}\right)}{\left(3+2\sqrt{2}\right)}$	$(\sqrt{2}-1)^2=2-\sqrt{2}-\sqrt{2}+1\left(=3-2\sqrt{2}\right)$ **or** $(\sqrt{2}+1)^2=2+\sqrt{2}+\sqrt{2}+1\left(=3+2\sqrt{2}\right)$ **or** $(\sqrt{2}-1)(\sqrt{2}+1)=2-\sqrt{2}+\sqrt{2}-1(=1)$			M1	oe ft $3-2\sqrt{2}$ method to rationalise **OR** expansion of $(\sqrt{2}-1)^2$ (accept $2-2\sqrt{2}+1$) **or** $(\sqrt{2}+1)^2$ (accept $2+2\sqrt{2}+1$) **or** $(\sqrt{2}-1)(\sqrt{2}+1)$
	eg $\frac{9+6\sqrt{2}+3\sqrt{8}+8}{9-6\sqrt{2}+6\sqrt{2}-8}$ **or** $\frac{9+12\sqrt{2}+8}{9-8}$ **or** $\frac{9+6\sqrt{2}+3\sqrt{8}+8}{1}$ **or** $\frac{9+12\sqrt{2}+8}{1}$				M1	dep on 2nd M1 correct expansion of brackets
			$17+\sqrt{288}$		A1	or $p=17$, $q=288$ answer from fully correct working with intermediate steps of working seen
						Total 4 marks

7	$\frac{12}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$ or $\frac{12}{\sqrt{2}-1}\times\frac{-\sqrt{2}-1}{-\sqrt{2}-1}$ **and** $4\sqrt{2}$ or $2\sqrt{8}$ or $\sqrt{32}$ oe		3	M1 for showing a correct method for rationalising the denominator **and** dealing with $\left(\sqrt{2}\right)^5$
	E.g. $12\sqrt{2}+12-4\sqrt{2}$ or $8\sqrt{2}+12$ $12\sqrt{2}+12-2\sqrt{8}$ or $12\sqrt{2}+12-\sqrt{32}$ oe			M1 dep expression must be in surd form
	E.g. $12\sqrt{2}(+12)-4\sqrt{2}=8\sqrt{2}(+12)=2\sqrt{4^2\times2}(+12)=2\sqrt{32}(+12)$ or $12\sqrt{2}(+12)-2\sqrt{8}=6\sqrt{8}(+12)-2\sqrt{8}=4\sqrt{8}(+12)=2\sqrt{4\times8}(+12)=2\sqrt{32}(+12)$ or $12\sqrt{2}(+12)-\sqrt{32}=3\sqrt{4^2\times2}(+12)-\sqrt{32}=2\sqrt{32}(+12)$ oe Note $8\sqrt{2}=2\sqrt{4^2\times2}$ or $2\sqrt{16\times2}$ or $\sqrt{32\times4}$ or $\sqrt{64\times2}$ $12\sqrt{2}=3\sqrt{4^2\times2}$ or $3\sqrt{16\times2}$ or $\sqrt{32\times9}$	Shown		A1 dep on M2 for showing working to given answer (they may dismiss the +12 and just deal with the surd part for this stage)
				Total 3 marks

8	$\sqrt{8}+4-\left(\sqrt{8}-4\right)(=8)$**and** $\sqrt{8}+4+\left(\sqrt{8}-4\right)\left(=2\sqrt{8}=4\sqrt{2}\right)$	$(a+b)(a-b)=a^2-b^2$ **and** $(\sqrt{8}+4)^2-(\sqrt{8}-4)^2$		3	M1 for correct substitutions into expression for $a+b$ **and** $a-b$ **or** expand the expression to get a^2-b^2 **and** substitute into this expression.
	$('8')\left('2\sqrt{8}'\right)$or $\sqrt{2048}$ or $16\sqrt{8}$ or $32\sqrt{2}$ or $8\sqrt{32}$ or $8\sqrt{8\times4}$ oe				M1 (dep M1)
			8		A1 (dep both M marks)
					Total 3 marks

9	eg $(AM =)\sqrt{x^2+(4x)^2}\ (= \sqrt{17x^2} = x\sqrt{17})$ oe **or** $(AM =)\sqrt{(0.5x)^2+(2x)^2}\ \left(=\sqrt{\frac{17}{4}x^2} = x\frac{\sqrt{17}}{2}\right)$ oe **or** $(AM =)\sqrt{20^2+5^2}\ (=\sqrt{425} = 5\sqrt{17})$ oe		5	M1	for a correct method to find AM as a numerical value or in algebraic form, must have brackets or recover
	Height of triangle eg $\sqrt{(2x)^2-x^2}\ (=\sqrt{3x^2} = x\sqrt{3})$ oe **or** $\sqrt{x^2-(0.5x)^2}\ \left(=\sqrt{\frac{3}{4}x^2} = x\frac{\sqrt{3}}{2}\right)$ oe **or** $\sqrt{10^2-5^2}\ (=\sqrt{75} = 5\sqrt{3})$ oe			M1	for a correct method to find height of equilateral triangle HJK as a numerical value or in algebraic form
	eg $\tan MAJ = \frac{\sqrt{3}+2}{\sqrt{17}}$ **or** $\tan MAJ = \frac{\frac{\sqrt{3}}{2}+1}{\frac{\sqrt{17}}{2}}$ **or** $\tan MAJ = \frac{5\sqrt{3}+10}{5\sqrt{17}}$			M1	for correct values for the correct angle (no algebra) **or** for $\tan MAJ$ is given numerically in the range 0.9 – 0.91
	eg $\frac{(\sqrt{3}+2)}{\sqrt{17}}\times\frac{\sqrt{17}}{\sqrt{17}}\left(=\frac{\sqrt{51}+2\sqrt{17}}{17}\right)$			M1	
		$\frac{\sqrt{68}+\sqrt{51}}{17}$		A1	or $\frac{\sqrt{51}+\sqrt{68}}{17}$
					Total 5 marks

10	eg $\frac{\sqrt{12}}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$		3	M1	rationalise denominator – award for seeing multiplication by $\frac{\sqrt{3}-2}{\sqrt{3}-2}$ or $\frac{-\sqrt{3}+2}{-\sqrt{3}+2}$
	eg $\frac{(\sqrt{36}-2\sqrt{12})}{3-4}$ **or** $\frac{6-2\sqrt{12}}{-1}$ **or** $-6+2\sqrt{12}$ **or** $\frac{6-4\sqrt{3}}{-1}$ **or** $-6+4\sqrt{3}$			M1	dep M1 correctly simplifying numerator and denominator. (denominator could be 3 – 4 or –1)
		$-6+\sqrt{48}$		A1	dep M2 must be in correct form (including $\sqrt{48}$) allow a = –6 and b = 48
					Total 3 marks

11	$\sqrt{3}x - x = 6+2\sqrt{3}$ oe or $x - x\sqrt{3} = -6-2\sqrt{3}$ (allow $-2\sqrt{9}$ or $-2(\sqrt{3})^2$ for -6 or $2\sqrt{9}$ or $2(\sqrt{3})^2$ for 6)		4	M1	expanding bracket and collecting terms. Condone one error
	$(x =)\ \frac{6+2\sqrt{3}}{\sqrt{3}-1}$ oe eg $\frac{-6-2\sqrt{3}}{1-\sqrt{3}}$			A1	oe must be a correct fraction with irrational numerator and denominator
	$(x =)\ \frac{(6+2\sqrt{3})}{(\sqrt{3}-1)}\times\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$ or $\frac{(6+2\sqrt{3})(\sqrt{3}+1)}{2}$ oe or $\frac{\left(6+2\sqrt{3}\right)}{\left(-1+\sqrt{3}\right)}\times\frac{\left(-1-\sqrt{3}\right)}{\left(-1-\sqrt{3}\right)}$ oe or $\frac{\left(-6-2\sqrt{3}\right)\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)}$ oe			M1	(indep) Multiplying the numerator and denominator of their fraction by $\sqrt{3}+1$ oe or showing 2 or –2 as the denominator and multiplying the numerator by $\sqrt{3}+1$ oe or rationalising **their** denominator, so long as it is of the form $p+q\sqrt{3}$ where p and q are non zero integers (condone missing brackets provided meaning is clear)
	Working required	$6+4\sqrt{3}$		A1	dep on M1A1M1 with no errors seen
					Total 4 marks

12 (a)		12	1	B1	
(b)	$\frac{5-\sqrt{18}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}$ **or** $\frac{5-\sqrt{18}}{1-\sqrt{2}}\times\frac{-1-\sqrt{2}}{-1-\sqrt{2}}$ oe		3	M1	Multiplying numerator and denominator by $1+\sqrt{2}$
	$\frac{5-\sqrt{36}+5\sqrt{2}-\sqrt{18}}{1+\sqrt{2}-\sqrt{2}-2}$ **or** $\frac{5-6-3\sqrt{2}+5\sqrt{2}}{-1}$ **or** $\frac{-5+6+3\sqrt{2}-5\sqrt{2}}{1}$ oe NB:allow $\sqrt{18}$ or $3\sqrt{2}$ $\sqrt{36}$ or 6 or $\sqrt{6}\sqrt{6}$			M1	Showing correct expansions (not necessarily as a fraction)
	working required	$1-2\sqrt{2}$		A1	dep on M2 (ie all stages of working must be shown convincingly) or for stating $a = 1$ and $b = -2$
					Total 4 marks

13	$\frac{2\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ or $\frac{2\sqrt{3}}{\sqrt{3}-1}\times\frac{-\sqrt{3}-1}{-\sqrt{3}-1}$		3	M1 for explicitly multiplying the numerator and the denominator by $\sqrt{3}+1$ or $-\sqrt{3}-1$
	$\frac{2\times3+2\sqrt{3}}{3-1}$ or $\frac{6+2\sqrt{3}}{3-1}$ or $\frac{6+2\sqrt{3}}{2}$ oe $\frac{-2\times3-2\sqrt{3}}{-3+1}$ or $\frac{-6-2\sqrt{3}}{-3+1}$ or $\frac{-6-2\sqrt{3}}{-2}$ oe			M1 dep on M1 (numerator expanded for 2 terms which need to be all correct and denominator may be 4 terms which need to be all correct)
	Working required	$3+\sqrt{3}$		A1 allow $\sqrt{3}+3$ (dep on M2)
				Total 3 marks

14	$(x^2 =)\ \frac{13+6\sqrt{5}}{2\sqrt{5}-3}$		4	M1	expression for x^2
	$\frac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3}$ or $\frac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\frac{-2\sqrt{5}-3}{-2\sqrt{5}-3}$			M1	dep on previous M1 showing a **correct** product to rationalise the denominator (must be correct x^2)
	eg $\frac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3}=\frac{99+44\sqrt{5}}{11}$ or eg $\frac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3}=\frac{26\sqrt{5}+39+60+18\sqrt{5}}{20-9}$ or eg $\frac{13+6\sqrt{5}}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3}=\frac{26\sqrt{5}+39+12(\sqrt{5})^2+18\sqrt{5}}{(2\sqrt{5})^2-3^2}$			M1	dep on previous M1 continuing the expansion of the product on the numerator and denominator – maybe one of these forms or a combination of forms
	Working required	$2+\sqrt{5}$		A1	dep on M3 accept $a = 2$, $b = 5$
					Total 4 marks